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Urban canopy scheme

David J. Thomson, Lois Huggett and Nicola I. Stebbing

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NAME

Numerical Atmospheric-Dispersion Modelling Environment

Met Office, FitzRoy Road, Exeter, Devon, EX1 3PB, United Kingdom

1 Introduction

There are two main ways in which urban areas influence the flow and dispersion. Firstly, the heat balance in urban areas is significantly perturbed due to reduced evaporation, altered radiation properties (e.g. albedo), heat storage in the city fabric and anthropogenic sources of heat. Secondly, the flow both within and above the urban canopy layer is directly altered by the obstacles (mainly buildings but also trees etc) which make up the urban canopy.

The urban canopy scheme in NAME doesn't address the first of these, with changes in heat balance generally treated, if at all, through the input meteorological data. However, when the effect of the obstacles on the mean flow and turbulence is not represented in the input meteorology, the urban canopy scheme can be used to apply a correction for this. In principle the scope of the parametrization is the canopy layer and roughness sublayer (i.e. the region below the 'inertial sublayer'), although, in the parametrization's current form, changes above the urban canopy layer are introduced only to allow a reasonably smooth match to the flow above. The scheme assumes the flow above the roughness sublayer reflects the effective roughness of the urban environment.

NAME uses a parameterized approach, where the urban canopy is treated as statistically homogeneous over 'neighbourhood' scales. Only basic information on the canopy is used to estimate the effect of the urban canopy on the vertical profiles of the mean flow and turbulence.

The scheme can be used with either the single site or NWP flow modules. When used with the NWP flow module, it is invoked only where the urban land use fraction exceeds 0.05.

The scheme is somewhat experimental with limited validation.

2 Urban Parameters

We use the following parameters to characterise the urban canopy.

- λ_p: The building plan area as a fraction of the horizontal area.
- λ_f: The building frontal area exposed to the flow as a fraction of the horizontal area.
- h_c: The mean urban canopy height - i.e. the mean building height.
- L_c: The building length scale - the average of the lengths and widths of buildings. For very closely spaced buildings it could be the average of a block of buildings, corresponding to the average canyon length.

If values are not provided, λ_p, λ_f and h_c are estimated from the urban fraction f using parametrizations by Bohnenstengel et al. (2011) and Bohnenstengel and Hendry (2016):

$$\lambda_p = 22.88f^6 - 59.47f^5 + 57.75f^4 - 25.11f^3 + 4.33f^2 + 0.19f$$

$$\lambda_f = 16.41f^6 - 41.86f^5 + 40.39f^4 - 17.76f^3 + 3.24f^2 + 0.06f$$

$$h_c = 167.409f^5 - 337.853f^4 + 247.813f^3 - 76.3678f^2 + 11.4832f + 4.48226.$$

When using single site flow, either λ_p, λ_f and h_c can all be provided via the input or the urban fraction can be provided. With NWP flow, λ_p, λ_f and h_c cannot be provided and the urban fraction is taken to be the urban land use fraction. L_c is taken to be 100m (this plays a minor role and there is currently little analysis available to make a more rational choice).

3 Mean wind profile

We assume a mean wind profile $U(z)$ of the following form:

$$U(z) = \begin{cases} (u_*/k)\log((z-d)/z_0) & \text{if } z \geq h_c \\ U(h_c)\exp(-(h_c - z)/l_{exp}) & \text{if } \hat{z} \leq z \leq h_c \\ (u_{*g}/k)\log((z+z_{0g})/z_{0g}) & \text{if } 0 \leq z \leq \hat{z}. \end{cases} \quad (1)$$

Here z is the height above the ground, u_* is the friction velocity based on the total drag on the atmosphere, $k = 0.4$ is von Karman's constant, d is the displacement height, z_0 is the effective roughness length of the urban environment, l_{exp} is the length scale over which the wind decays exponentially as we move downwards through the canopy, $\hat{z}$ is the matching height between the lower two parts of the profile, u_{*g} is the friction velocity based on the drag on ground at the bottom of the canopy, and z_{0g} is the roughness length of the ground ignoring the effect of the buildings, taken to be 0.1 m. For displacement height d we follow Raupach (1994):

$$\frac{d}{h_c} = 1 - \frac{1 - \exp(-\sqrt{15\lambda_f})}{\sqrt{15\lambda_f}}.$$

For l_{exp} we follow Macdonald (2000):

$$l_{exp} = h_c/(9.6\lambda_f).$$

(1) is based on ideas presented by Macdonald (2000) and Coceal and Belcher (2004) and in particular on ideas from mixing length theory which are discussed further in appendix A. The three regimes correspond to (i) standard surface layer theory, applied right down to the top of the canopy, (ii) a regime where the velocity decays exponentially as one moves down through the canopy and where the mixing length is roughly constant, (iii) and a regime where we expect a standard surface layer to develop close to the ground. For the exponential regime, various expressions for l_{exp} are possible and are discussed in appendix A.

To determine $\hat{z}$, we equate $U/(dU/dz)$ from the two forms at $\hat{z}$. This gives

$$l_{exp} = (\hat{z} + z_{0g}) \log\left(\frac{\hat{z} + z_{0g}}{z_{0g}}\right). \quad (2)$$

This equation can be solved for $\hat{z}$ by using the iterative calculation

$$\hat{z}_1 = \max(l_{exp}, z_{0g}e) - z_{0g}; \quad \hat{z}_{n+1} = \frac{l_{exp} + \hat{z}_n + z_{0g}}{\log((\hat{z}_n + z_{0g})/z_{0g}) + 1} - z_{0g}.$$

Here $\hat{z}_1$ has been chosen to be a value that we know is greater than or equal to the true solution and this is followed by a Newton-Raphson iteration

$$\hat{z}_{n+1} = \hat{z}_n - \frac{f(\hat{z}_n) - l_{exp}}{f'(\hat{z}_n)}$$

where $f(\hat{z}) \equiv (\hat{z} + z_{0g}) \log\left(\frac{\hat{z}+z_{0g}}{z_{0g}}\right)$ is the right hand side of (2). Because the derivative of $f(\hat{z})$ is positive and increases with $\hat{z}$ it is clear that this must converge to the solution of (2) (see figure 1). u_* and u_{*g} can then be expressed in terms of $U(h_c)$ using continuity at $z = h_c$ and $z = \hat{z}$

$$\frac{u_*}{k} = U(h_c)/\log((h_c - d)/z_0)$$

$$\frac{u_{*g}}{k} = \frac{U(h_c)\exp(-(h_c - \hat{z})/l_{exp})}{\log((\hat{z} + z_{0g})/z_{0g})}.$$

In fact, because of the way we match to the flow above h_c (see below), we don't actually need the formula for u_*. u_* is needed for estimating turbulence parameters (see next section), but here we use the value of u_* as provided by the single site and NWP flow modules (prior to any adjustment for the canopy).

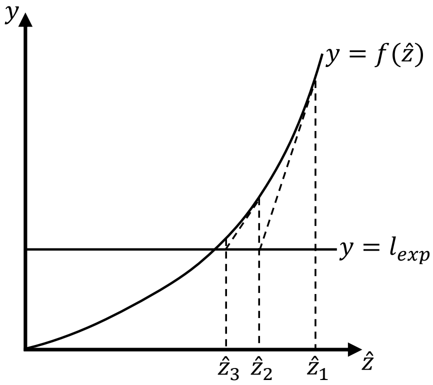

Figure 1: Illustration of the Newton-Raphson iteration for $\hat{z}$.

To use (1) we need to determine $U(h_c)$. There is also a need to match to the mean flow without the canopy modification, $U_{\text{no canopy}}(z)$, and this requires some adjustments to $U(z)$ in (1) for $z \geq h_c$. The shape of the profile $U_{\text{no canopy}}(z)$ may well be affected by stability and we don't want to lose any information on this that is contained in the profile, at least for $z \geq h_c$. In addition, $U_{\text{no canopy}}(z)$, as provided by the single site and NWP flow modules (prior to any adjustment for the canopy), is designed to go to zero at $z = 0$ and so, in neutral conditions, is proportional to $\log((z + z_0)/z_0)$ and has a different shape to the shape of profile (1) for $z \geq h_c$. These different shapes mean it is best to match some way above h_c. We achieve this by setting

$$U(3h_c) = U_{\text{no canopy}}(3h_c)$$

and

$$U(h_c) = U_{\text{no canopy}}(h_c)\frac{\log((h_c - d)/z_0)}{\log((h_c + z_0)/z_0)}.$$

The expression for $U(h_c)$ can be thought of as preserving the value of u_* implied by $U_{\text{no canopy}}$ and applying it in (1). In between h_c and $3h_c$ we use $U(z) = U_{\text{no canopy}}(z) \times$ canopy factor where canopy factor is a linear function of $\frac{\log((z-d)/z_0)}{\log((z+z_0)/z_0)}$ and interpolates between $\frac{\log((h_c-d)/z_0)}{\log((h_c+z_0)/z_0)}$ at $z = h_c$ and 1 at $z = 3h_c$.

4 Turbulence levels

Above the urban canopy, the usual formulae for $\sigma_{u,v,w}$ are used, but with $z + z_0$ replaced by $z - d$.

Within the canopy, turbulence levels are estimated by equating the rate of turbulent kinetic energy (TKE) production by shear and the rate of dissipation of TKE (we assume shear production dominates over buoyant production within the canopy). Assuming a mixing length l, this gives

$$\frac{(TKE)^{3/2}}{l} \propto l^2(dU/dz)^3$$

which yields

$$TKE \propto l^2(dU/dz)^2.$$

In the region with the exponentially varying mean flow ($\hat{z} \leq z \leq h_c$), we expect the mixing length to be roughly constant. Balancing production and dissipation then gives

$$TKE \propto \frac{l^2}{l_{exp}^2}U(z)^2$$

Because the stress is continuous across $z = h_c$ and $\sigma_{u,v,w\ \text{no canopy}}(z)$ does not generally vary much across the canopy, we can represent this by taking

$$\sigma_{u,v,w} = \sigma_{u,v,w\ \text{no canopy}}(z \approx 0)\frac{U(z)}{U(h_c)}.$$

In the log layer below the exponential decay, we expect

$$\sigma_{u,v,w} = \sigma_{u,v,w\ \text{no canopy}}(z \approx 0)\frac{u_{*g}}{u_*}$$

where we have again used the fact that $\sigma_{u,v,w\ \text{no canopy}}(z)$ does not generally vary much across the canopy.

A suitable transition between these forms is achieved (for $z \leq h_c$) by taking

$$\sigma_{u,v,w} = \sigma_{u,v,w\ \text{no canopy}}(z) \times \max\left(\exp\left(-\frac{h_c - z}{l_{exp}}\right), \frac{u_{*g}}{u_*}\right).$$

Here it is convenient to use $\sigma_{u,v,w\ \text{no canopy}}(z)$ which is defined here as being based on the usual formulae in the absence of canopy modifications, but with $z + z_0$ replaced by $z - d$.

The dissipation rate can be estimated as

$$\varepsilon = \begin{cases} u_*^3/k(z-d) & \text{if above canopy} \\ u_{*local}^3/k(h_c-d) & \text{if in exponentially decaying layer} \\ u_{*g}^3/k(z+z_{0g}) & \text{if in near surface log layer.} \end{cases}$$

To ensure continuity and match with above canopy values, we implement this by replacing the neutral stability surface layer part of $\varepsilon_{\text{no canopy}}$, i.e. the $u_*^3/k(z+z_0)$ term in $\varepsilon_{\text{no canopy}}$, with

$$\begin{cases} u_*^3/k(z-d) & \text{if } z \geq h_c \\ \max\left(\frac{u_*^3}{k(h_c-d)}\exp\left(-3\frac{h_c-z}{l_{exp}}\right), \frac{u_{*g}^3}{k(z+z_{0g})}\right) & \text{if } z \leq h_c. \end{cases}$$

Dispersive stresses, i.e. second order moments arising from variations in the time averaged flow between different horizontal positions within the canopy, can play an important role in dispersion. For narrow street canyons forming a square grid of streets orientated at 45 degrees to the wind we expect $\sigma_v = U$, $\sigma_u = 0$. These are the spatial standard deviations of the wind components that arise from a uniform flow down each street when the magnitude of the spatially averaged wind vector is U. For streets at 0 degrees and 90 degrees to the wind we expect $\sigma_v = 0$, $\sigma_u = U$. These are the spatial standard deviations of the wind components that arise from a uniform flow down each wind aligned street with zero flow in the cross wind streets. Heuristically we estimate that, averaging over canyon directions, $\sigma_u^2 = \sigma_v^2 = U^2/2$. This is also the result if we assume the flow in each canyon is proportional to the component of the above canyon wind in the canyon direction (as found by Dobre et al. (2005) in the DAPPLE experiment) and we average more systematically over all possible canyon orientations. For sparse buildings we expect $\sigma_u \sim \sigma_v \sim U$ near the building but ~ 0 elsewhere. We cover both cases with dispersive velocity variances of $\sigma_u^2 = \sigma_v^2 = U^2\lambda_p/2$. These are added to the turbulent velocity variances. The time scale associated with these 'dispersive stresses' is estimated as L_c/U.

Appendix A: Notes on mixing length models of the canopy

We consider a canopy whose properties are sufficiently horizontally homogeneous that the flow at heights of order the canopy height is in equilibrium with the surface and hence that the dynamics can be represented with a 1-D model. As usual for equilibrium near surface flows over rough surfaces, we assume that the momentum balance is dominated by the drag on the roughness elements and by transfer of momentum from above, with pressure gradient and Coriolis forces being negligible. Following Coceal and Belcher (2004) (in the context of using a mixing length model to estimate the mean flow) we neglect the 'dispersive' stresses, i.e. the stresses arising from spatial fluctuations in the time mean flow We also neglect thermal effects, consistent with the likely dominance of mechanically generated turbulence within the canopy layer except in very light winds.

In some of the following we allow for the possibility that λ_p and λ_f vary with z, although we don't currently make use of this in the urban scheme. The definition of $\lambda_p(z)$ is straightforward (the plan area fraction occupied by buildings at height z) and we define $\lambda_f(z)$ as the frontal area in a height range $[z, z+dz]$ scaled up by a factor h_c/dz (so that the definition agrees with the normal definition when λ_f doesn't vary with z). If cumulative frontal area up to height z is $\Lambda_f(z)$, $\lambda_f(z) = h_c d\Lambda_f/dz$.

There is a choice over whether the primary variable is taken to be the mean flow velocity at height z averaged over the area not occupied by buildings – denoted by $U(z)$ – or is the area integral of the the velocity divided by the plan area – which equals $(1-\lambda_p(z))U(z)$. The latter has the advantage that the momentum is easily related to its integral but the former is perhaps more intuitive and is adopted here. The latter is also formally continuous at building tops (jumps in λ_p) while the former isn't (because of the no slip boundary on the building tops). However, this apparent advantage is less relevant in practice because the boundary layer on building tops is likely to be thin. If this thin layer is excluded from the averaging in calculating U then the former definition may actually in practice be roughly continuous while the latter won't be. The thin shear layer over the buildings doesn't then contribute to $U(z)$ and the associated stresses do not appear in the flow description. Consistent with this, the frictional drag on the building roofs is neglected. We will follow this approach here. Note that, although not relevant for the 1-D horizontally homogeneous and stationary situation discussed here, this approach implies that the mass conservation equation needs modification as the mass flux associated with $U(z)$ requires a factor of $1-\lambda_p$ to be added.

The momentum equation within the canopy can be formulated by equating the divergence of the shear stress to the drag on the canopy elements, and this is often done using a mixing length model. Macdonald (2000) neglected the reduction in the volume of air (the factor $1-\lambda_p$) within the canopy which the drag acts on and so formulated the momentum equation as

$$\frac{d}{dz}\left(l^2\left(\frac{dU}{dz}\right)^2\right) = \frac{c_d(z)\lambda_f(z)}{2h_c}U^2.$$

Here $c_d(z)$ is the sectional drag coefficient and l is the mixing length. Coceal and Belcher (2004) corrected for the fact that the building drag is acting only on the air outside the buildings and assumed λ_p and λ_f constant with z (up to h_c) to obtain

$$\frac{d}{dz}\left(l^2\left(\frac{dU}{dz}\right)^2\right) = \frac{c_d(z)\lambda_f}{2h_c(1-\lambda_p)}U^2.$$

If λ_p and λ_f vary with z, one should balance the divergence of the horizontally integrated stress against the drag, with the horizontal integral being over the area outside the buildings. This leads to

$$\frac{d}{dz}\left((1-\lambda_p(z))l^2\left(\frac{dU}{dz}\right)^2\right) = \frac{c_d(z)\lambda_f(z)}{2h_c}U^2. \quad (3)$$

Note that, even for the case of constant λ_p and λ_f, the change in the position of the $1-\lambda_p$ factor in (3) relative to Coceal and Belcher does have an effect on the transition between the in canopy and above canopy flow and, as a consequence, the implied effective z_0. These equations have neglected the frictional drag on the building roofs and perhaps (depending on precisely how c_d is defined) the frictional drag on the building walls.

Equation (3) is perhaps best regarded as solved by taking a given surface stress at the ground $z=0$ and $U(0)=0$, integrating upwards, and then rescaling the stress and velocity profiles to match a given value of $U(z)$ above the canopy. To show this is well defined we can envisage solving two coupled first order ODEs:

$$\frac{d}{dz}\tau = \frac{c_d(z)\lambda_f(z)}{2h_c}U^2 \quad \text{and} \quad \frac{d}{dz}U = \left(\frac{\tau}{(1-\lambda_p(z))l^2}\right)^{1/2}.$$

Specifying the mixing length is difficult. Coceal and Belcher (2004) represent l as

$$l = \begin{cases} k(z-d) & \text{if } z \geq h_c \\ k(h_c-d) & \text{if } h_c-d \ll z \leq h_c \\ k(z+z_{0g}) & \text{if } z \ll h_c-d \end{cases} \quad (4)$$

with some match between these regimes. Note here we have defined l near the ground so that a log profile of the form $\log((z+z_{0g})/z_{0g})$ emerges. For surfaces which we model through a roughness length rather than a canopy, it is convenient to regard the origin of z as being the height at which the extrapolated log profile goes to zero. (4 means the wind profile above h_c will be proportional to $\log((z-d)/z_0)$ for some z_0. While formally one would expect this only well above h_c there is evidence to suggest it can be a reasonable approximation for the horizontally averaged profile in the roughness sublayer above the canopy layer (see e.g. Best et al. (2008, p. 53)). One could debate whether this should be $\log((z+z_{0g}-d)/z_0)$ consistent with $\log((z+z_{0g})/z_{0g})$, but we expect z_{0g} to be within the uncertainty in d.

This form of mixing length really only makes sense if the canopy doesn't have multiple scales such as dense buildings of height h_1 and much sparser but taller buildings of height h_2 and it is hard to envisage how l might be chosen for arbitrary canopies. In the following we will assume λ_f, λ_p and $c_d(z)$ are uniform with height up to $z=h_c$ and zero above that height. For this case d can be chosen as in e.g. Raupach (1994) or Macdonald et al. (1998). Grimmond and Oke (1999) compared various methods of estimating d against a range of data from urban areas and concluded that, of the methods using a small number of bulk parameters, the methods of Raupach (1994) and Macdonald et al. (1998) seemed preferable, although with a large scatter in the results.

The depth to which the flow penetrates inside the canopy is an important parameter. Suppose $l \simeq k(h_c-d)$ in a layer below $z=h_c$ (as implied by (4)). In this layer (3) has a solution $U=U(h_c)\exp(-(h_c-z)/l_{exp})$ for $z \leq h_c$ with the exponential decay length scale l_{exp} equal to $[4l^2h_c(1-\lambda_p)/c_d\lambda_f]^{1/3}$ ((Macdonald, 2000; Coceal and Belcher, 2004)). This isn't necessily the correct solution as that will depend on what happens at the boundaries of the layer. However, if the canopy is 'deep' so that the velocity decays substantially in this layer, then the small velocity at the bottom of the layer should make it easy to satisfy the no-slip boundary condition at the surface and the long exponential decay should allow the precise boundary condition at the bottom of the layer to be 'forgotten' in integrating upwards through the layer. This requires $h_c-z_1 \gg l_{exp}$ where z_1 is the bottom of the layer. Provided λ_f, λ_p and c_d are not too extreme this can be expressed as $h_c-z_1 \gg h_c-d$.

To demonstrate this a bit more rigorously, write the equation (3) as $dU'^3/dU = 3U^2/l_{exp}^3$ (where $U' \equiv dU/dz$). This can be integrated to get $U'^3 - U'(z_1)^3 = (U^3-U^3(z_1))/l_{exp}^3$ which can be integrated again to get

$$\frac{z-z_1}{l_{exp}} = \int_{U(z_1)}^{U(z)} \frac{dU}{(U^3+l_{exp}^3U'(z_1)^3-U^3(z_1))^{1/3}}.$$

Putting $C = l_{exp}^3 U'(z_1)^3/U^3(z_1) - 1$, this can be expressed as

$$\frac{z - z_1}{l_{exp}} = \int_{U(z_1)}^{U(z)} \frac{dU}{(U^3 + CU^3(z_1))^{1/3}} = \int_{U(z_1)}^{U(z)} dU \left(\frac{1}{U} - \frac{(1 + CU^3(z_1)/U^3)^{1/3} - 1}{(1 + CU^3(z_1)/U^3)^{1/3} U} \right)$$

$$= \log \frac{U(z)}{U(z_1)} - \int_1^{U(z)/U(z_1)} dV \frac{(1 + C/V^3)^{1/3} - 1}{(1 + C/V^3)^{1/3} V}.$$

As $U(z)/U(z_1) \to \infty$, the last term approaches a constant (depending on C) and so $U(z) \propto \exp(z/l_{exp})$.

Note the expression for l_{exp} agrees with Macdonald (2000) equation (8) if we ignore the $1-\lambda_p$ factor. To compare further with Macdonald (2000), it is useful to express l_{exp} in terms of the bulk drag coefficient C_D (based on the velocity $U(h_c)$). Now $C_D = c_d l_{exp}/2h_c$ so $l_{exp} = [2l^2(1 - \lambda_p)/C_D \lambda_f]^{1/2}$. This agrees with Macdonald (2000) equation (11), ignoring the $1 - \lambda_p$ factor. Macdonald also derived an empirical expression for l_{exp} based on wind tunnel data with cubic obstacles which takes the form $l_{exp} = h_c/(9.6\lambda_f)$.

Appendix B: Notes on dispersive stress calculation for narrow street canyons

Here we provide more detail on the dispersive stress calculation for the case of narrow street canyons. We first consider a regular array of cuboid buildings. We neglect the cross street flow and the flow in the street intersections and we use Dobre et al. (2005)'s model where the wind along the street is proportional to the component of the wind above the urban canopy in the along street direction. The wind vector within the streets is illustrated in figure 2. Using the notation in the figure and writing $c \equiv \cos\theta$ and $s \equiv \sin\theta$, the wind vector is then

$$(\hat{u}c^2, \hat{u}cs) \text{ in area fraction } \tfrac{L}{L+W}$$
$$(\hat{u}s^2, -\hat{u}cs) \text{ in area fraction } \tfrac{W}{L+W}.$$

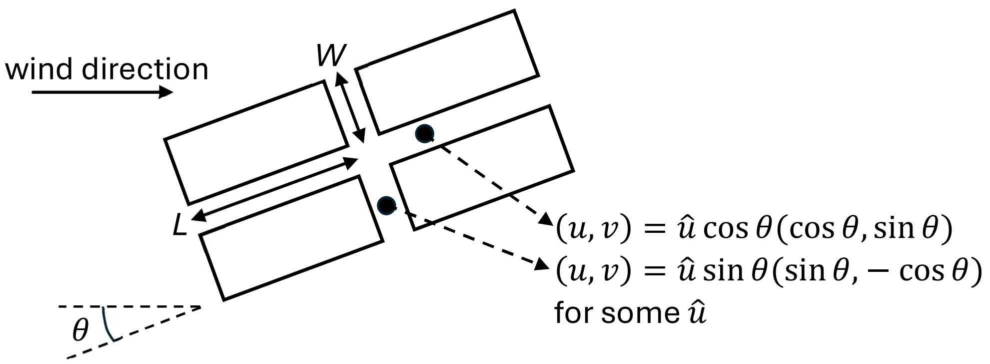

Figure 2: Illustration of flow through a regular array of buildings with narrow, and similar, street canyons. The wind vector due to along street flow at two points is shown, using Dobre et al. (2005)'s model where the wind along the street is proportional to the component of the wind above the urban canopy in the along street direction. $\hat{u}$ is unknown and will vary with height but is otherwise the same at the two locations. The (u, v) vector components are defined as along and across the above canopy wind direction.

We note that $c^2 - 1/2$, $s^2 - 1/2$ and cs are harmonic functions with mean zero and amplitude 1/2. Hence, if θ is uniformly distributed, these have mean zero and variance 1/8. If we use $\langle(-)\rangle$ to denote averages over θ it follows that

$$\langle c^2 \rangle = \langle s^2 \rangle = 1/2, \quad \langle cs \rangle = 0, \quad \langle (c^2 - 1/2)^2 \rangle = \langle (s^2 - 1/2)^2 \rangle = \langle c^2 s^2 \rangle = 1/8$$

and, by symmetry,

$$\langle c^3 s \rangle = \langle cs^3 \rangle = 0.$$

The horizontally averaged flow (outside the buildings) is

$$\frac{L}{L+W}(\hat{u}c^2, \hat{u}cs) + \frac{W}{L+W}(\hat{u}s^2, -\hat{u}cs)$$

and so the mean flow averaged both horizontally and over θ is

$$\frac{L}{L+W}(\hat{u}\langle c^2\rangle, \hat{u}\langle cs\rangle) + \frac{W}{L+W}(\hat{u}\langle s^2\rangle, -\hat{u}\langle cs\rangle) = (\hat{u}/2, 0).$$

The variances and covariances (over both horizontal variations and fluctuations in θ) are given by

$$\text{variance}(u) = \frac{L}{L+W}\hat{u}^2\langle(c^2-1/2)^2\rangle + \frac{W}{L+W}\hat{u}^2\langle(s^2-1/2)^2\rangle = \hat{u}^2/8,$$

$$\text{variance}(v) = \frac{L}{L+W}\hat{u}^2\langle c^2s^2\rangle + \frac{W}{L+W}\hat{u}^2\langle c^2s^2\rangle = \hat{u}^2/8$$

and

$$\text{covariance}(u,v) = \frac{L}{L+W}\hat{u}^2\langle c^3s\rangle + \frac{W}{L+W}\hat{u}^2\langle cs^3\rangle = 0.$$

Identifying U with the mean flow $\hat{u}/2$ gives variance(u) = variance$(v) = U^2/2$.

The above calculates the mean flow and dispersive stress for an ensemble of flows where, as well as an ensemble of turbulent fluctuations being included, we include flows with different street orientations. If θ isn't known it may be appropriate to use this larger ensemble, although, were θ to be known, one could in principle use a mean flow specific to θ and a smaller variance by considering the sub-ensemble with fixed θ. We note that, for $L = W$, the mean flow and variance do not actually depend on θ and so the variance in the larger ensemble is then the same as that in the sub-ensembles. One can also interpret the averaging over street directions as applicable to neighbourhoods with many different, and roughly uniformly distributed, street directions in which case the averaging over θ can be regarded as part of the horizontal averaging.

This last remark suggests a simpler derivation of the above results which doesn't assume a regular building geometry, assuming we are happy to average over orientations of the geometry or the geometry is roughly statistically isotropic. We can consider a single street canyon and average over the angle of the canyon to the flow. The calculation for this case amounts to putting $W = 0$ in the above and gives the same result as before.

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